

Universal quantifiers in math and late language development in school-aged children

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Language development research of the past 20 years has revealed that many aspects of semantics and pragmatics are acquired late, not until the school years. In the classroom, word problems in mathematics make extensive use of language. Here, we explore whether math language includes late acquired semantic constructions. As a first step, we focus on two Dutch quantifiers *alle* ‘all’ and *elke* ‘each’. Using the semantic notion of distributivity and applying insights from acquisition studies, we explore word problems in second-grade math books in Dutch. We find problems that target quantifier interpretations that may not be fully acquired by the age of 7–8, or are ambiguous even for adult speakers. We then formulate open questions that are triggered by this exploration, offering preliminary results. This study makes a case for interdisciplinary collaboration across semantics, language development and education to investigate whether math problems match school children’s acquisition path.

Keywords: semantics, universal quantifiers, distributivity, first language acquisition, late acquisition, mathematics education, word problems

1. Introduction

In the past 20 years many studies of language development during the school age years (5–12) have shown that subtle distinctions in the realm of semantics and pragmatics are acquired late, after children start attending school (Tsimpli 2014, Schulz & Grimm 2019, Schulz 2024). At school, education materials make extensive use of language. The fact that typical language development continues well into the school years raises the following questions: Do semantic and pragmatic phenomena that are acquired late, appear in school materials? If so, can children’s non-adultlike interpretation affect their understanding of these materials? Apply-

ing linguistic knowledge from sentence semantics and first language acquisition to word problems in math, we explore if there exist problems where a non-adultlike interpretation would lead to the “wrong” answer.

As a case in point, we analyzed word problems from mathematics textbooks, focusing on Dutch quantifiers *alle* ‘all’ and *elke* ‘each’. We selected these quantifiers because we expected them to appear in word problems that target multiplication and division. However, 7-to-8-year-olds have shown non-adultlike interpretations concerning the semantic notion of distributivity. For example, in a problem such as “*Farmer Tom has 4 meadows. In each meadow, there are 6 cows. How many cows does farmer Tom have?*”, the target distributive interpretation is to evenly *distribute* cows over meadows in such a way that each of the 4 meadows has 6 cows, making 24 in total. Children, on the other hand, also accept a non-distributive cumulative situation for sentences with *each*, with 4 meadows and 6 cows *in total* (for example, 2 meadows with 2 cows and 2 meadows with 1 cow) (Drozd et al. 2017). *Alle*, moreover, is ambiguous in the adult language between a distributive and a non-distributive interpretation. The answer keys for math problems, however, list only one correct answer, which raises the question how to know what the targeted interpretation is.

This paper offers an exploratory analysis of Dutch word problems with universal quantifiers from the semantic perspective of a Dutch learner at school age. After reviewing the acquisition path of Dutch quantifiers to identify non-adultlike interpretations (Section 2), we provide real-life examples of math problems containing the quantifiers *alle* and *elke*. We compare multiple interpretations of each problem, pointing out the adult-like interpretation alongside the learner’s non-adultlike one (Section 3). We discuss the gaps in our current knowledge (Section 4), leading to a novel research agenda with societal relevance aimed at finding out to what extent children struggle with math language in word problems (Section 5). We argue for the need of interdisciplinary research bridging linguistics and education to make sure that school language matches the acquisition path.

2. Semantics and L1 studies of distributive and collective readings

In sentences with a noun phrase that denotes a plurality, including definite plurals (e.g., *the children*), numerals (e.g., *fifteen children*) and quantified NPs (e.g., *most of the children*), the interpretation of the noun phrase can be distributive – when the predicate applies to each atomic member of the set separately and individually, or collective – when the predicate applies to the plurality as a group (Champollion 2021). These readings can be triggered by the lexical meaning of

the verbal predicate (Link 1983). Predicates such as *laugh* enforce a distributive reading (1a), whereas verbs such as *gather* require a collective reading (1b). Many other verbs, including for instance *sing*, do not encode one of these intrinsic readings; they are compatible with distributive and collective readings alike (1c).

- (1) De / vijftien / meeste kinderen...
 'the / fifteen / most children...'
 a. ...lachten.
 'laughed.'
 b. ...verzamelden zich.
 'gathered.'
 c. ...zongen.
 'sang.'

In transitive sentences with a plural-denoting NP, the notions of distributivity and collectivity determine the nature of the relation between subject and object NP. The distributive verb *swallow* determines a one-to-one relation between the plural subject and the indefinite singular object in (2a): there are multiple vitamin pills, one for each child. The collective verb *surround* on the other hand involves a one-to-many relation between the two NPs in (2b): there is one tree and a group of children jointly surrounding it. Most verbs, however, are not specified for this meaning feature. (2c) with *climb* is just as good a description of a one-to-one relation (multiple trees, individual children climbing their own), or a one-to-many relation (the group together climbing the same tree), thus allowing ambiguity between a distributive and collective reading. (2c) can furthermore describe a mixed situation where some children jointly climbed the same tree, while others climbed their own, individual trees. This is called a cumulative reading (Scha 1981).

- (2) De / vijftien / meeste kinderen...
 'the / fifteen / most children...'
 a. ...slikten een vitaminepil in.
 'swallowed a vitamin pill.'
 b. ...omringenden een grote boom.
 'surrounded a big tree.'
 c. ...klommen in een boom.
 'climbed a tree.'

Distributive or collective meaning is not only marked on certain verbs, (1a)–(b) and (2a)–(b), but can also be lexically encoded in quantifiers. While *all* in (3) is compatible with a distributive and a collective reading, *each* and *every* in (4) are specified for distributivity. They are compatible with a distributive verb, (4a), and cannot combine with collective verbs, (4b) (Vendler 1962). With verbs that are

unspecified for distributivity such as *climb*, only the distributive reading obtains: (4c) describes a one-to-one relation between children and trees (multiple trees, one for each child), but not a group reading with a one-to-many relation (one tree for all children together).

- (3) Alle kinderen klommen in een boom.
'all children climbed a tree'
- (4) Elk / ieder kind...
'each / every child...'
 - a. ...lachte / slikte een vitaminepil in.
'laughed / swallowed a vitamin pill'
 - b. *...verzamelde zich / omringde een boom.
'gathered / surrounded a tree'
 - c. ...klom in een boom.
'climbed a tree'

In addition to the notion of distributivity encoded in *each* and *every*, but not in *all*, there is another meaning dimension that the three universal quantifiers share: exhaustivity (Syrett 2019). Sentences with a universal quantifier are only true if the predicate applies to each element of the set under discussion, e.g., if every-one in the set of the children are tree climbers (3), (4c). Thus, the three universal quantifiers are not equal: while all three of them are exhaustive, *each* and *every* are distributive, and *all* is unspecified. This subtle difference poses a challenge for children who are learning the meanings of these words.

The two meaning components of universal quantifiers, exhaustivity and distributivity, have been extensively investigated in acquisition studies, and several non-adult-like interpretation patterns have been discovered. Most focused on the exhaustivity property (see Brooks & Parshina 2019, for a review). There is an initial stage of underexhaustive pairing in which 3-to-4-year-olds do not verify if the predicate applies to the whole set, allowing extra agents for sentences like (3) and (4c) (some non-climbing children). By age 5 or 6, preschoolers correctly reject a situation with extra agents, revealing knowledge of exhaustivity. However, the same children reject situations with extra objects (some trees without any children in them), which, in the adult verification process, are irrelevant for assessing the universally quantified subject. In this stage children only accept a symmetrical situation where all agents and objects are paired (Inhelder & Piaget 1964 and much subsequent work). This error pattern has been attested up to age 7 (Aravind et al. 2017). So, even though children have typically acquired exhaustivity before they start school, pictures with extra objects still seem to "trick" them into an overly restrictive interpretation (Smits 2010).

As for distributivity, non-adultlike readings have been found up to age 10 (see Syrett 2019, for a review). In this stage children are overly liberal with distributive quantifiers *each* and *every* as compared to adults: they correctly accept them for distributive situations, but also, incorrectly, accept them for non-distributive situations. Here, we review in some detail studies in which the visual stimuli satisfied exhaustivity and varied collective and distributive situations (without any extra agents or objects). The experimental manipulations targeted the relation between agents and objects: one-to-one, one-to-many or mixed.

The first observation of children's more permissive interpretation of *each* in English comes from Brooks and Braine (1996) who used pictures in which agents and objects were exhaustively paired, either in a distributive, one-to-one relation or a collective, one-to-many relation. In a forced-choice picture-selection task (their Experiment 2), adults never selected the collective picture for sentences such as *each boy is building a boat*, but children ages 4 to 7 regularly did (between 24–44%). A picture-selection task only reveals preferred interpretations and does not allow assessing participants' full range of possible meanings. Later studies replicated children's overly liberal interpretation using other methods and languages. For instance, in a picture-verification task, the collective reading was widely accepted for floating *each* (*two boys each pushed a car*) by English 4-year-olds, and to some extent also by adults (32%) (Syrett & Musolino 2013). Similarly, the Italian distributive quantifier *ciascun* 'each' was only accepted for distributive situations by Italian adults, but Italian children up to age 10 also accepted collective situations (between 26–89%, Pagliarini et al. 2012). The conclusion is that children are overly permissive: *each* and *every* seem truly ambiguous and allowed in distributive as well as collective situations by children, but not adults.

Studies on distributivity in Dutch children (and adults) revealed somewhat different interpretation patterns. Rouweler and Hollebrandse (2015) investigated three types of situations: distributive, collective and cumulative for sentences with *alle* and *elke* (*all boys are washing a boat* and *every boy is washing a boat*). In the cumulative or 'mixed' interpretation, two boys are washing the same boat while another boy is washing a separate boat. In the picture-selection task (their Experiment 2), adults and 5-year-olds chose the distributive situation for *elke*, suggesting sensitivity to its distributive meaning. Surprisingly, children significantly preferred the distributive interpretation for *alle* (84%), whereas adults did not have a significant preference (40% collective, 60% distributive). In a picture-verification task (their Experiment 1), *alle* was however accepted for all three types of situations by adults and children alike and *elke* was accepted for the distributive situation by all participants. The crucial conditions were *elke* in the collective and cumulative conditions. In line with prior findings in English and Italian, *elke* was accepted by the children for collective (around 70%) and cumulative (almost

80%) situations. But in contrast with these studies, Dutch adults also accepted these situations for *elke* around half of the time. De Koster and colleagues replicated the ambiguity of *elke* and *iedere* ‘every’ in Dutch adults in a series of studies employing various methods, a truth-value judgment task (de Koster et al. 2017, 2018, 2020a) and a covered-box paradigm (de Koster et al. 2020b). There were some differences in acceptance rates of the collective reading in children ages 4 to 11 (across studies between 17–86%) and adults (across studies between 22–50%). Nevertheless, the conclusion is that Dutch *elke* and *iedere* are accepted in non-distributive interpretations for adults, and therefore, it is not surprising that children allow for a collective or cumulative reading in addition to a distributive one.¹

Given the aim of the current paper, the analysis of word problems that target multiplication (Section 4), Drozd et al.’s (2017) study on distributivity is highly relevant. All studies discussed so far tested sentences with a universally quantified subject and a singular indefinite NP. Drozd and colleagues instead used sentences with a plural object with a numeral, for example, *every cowboy is pulling two horses*. They compared situations where the number of cowboys and horses was either multiplied, showing a distributive reading (3 cowboys, each pulling 2 horses, so 6 horses in total), or not, presenting a cumulative reading (3 cowboys together pulling 2 horses in total, with the middle cowboy pulling 2 horses and the other two each pulling 1). English adults mostly rejected this cumulative reading (11% acceptance), but children ages 5 to 8 accepted it quite often (between 35–51%) (Drozd et al. 2017). The conclusion is that children have a non-adult-like reading: over-acceptance of the cumulative reading for *every*.²

To summarize, there is evidence that English *each* and *every* are ambiguous in school-aged children (Brooks & Braine 1996, Drozd et al. 2017). While Dutch children prefer a distributive interpretation for *elke*, they accept the collective and cumulative interpretation as well (Rouweler & Hollebrandse 2015, De Koster et al. 2018, unpublished study in footnote 2).

Seeing that school children accept universal quantifiers in non-adultlike interpretations, the following question arises: How might a non-adult-like inter-

1. An insightful comment from one of our reviewers states that in experiments like the one reported on in Rouweler & Hollebrandse (2015), choosing the collective or cumulative picture does not necessarily show that the participant does not have a distributive interpretation for *elke*. The cumulative and collective picture, in which multiple boys are washing the same boat, can still be described fully distributively: boy 1 is washing a boat, boy 2 is washing a boat, and boy 3 is washing a boat. Drozd et al.’s (2017) study does not face the same criticism, which is why we turn to it next.

2. Ken Drozd led a crosslinguistic follow-up study in 12 child languages, to which Angeliek van Hout contributed findings showing overly permissive interpretation of Dutch *elke*. This work has not been written up.

pretation affect their interpretation of word problems in math? In light of the studies discussed above, Dutch children between 6 and 9 may interpret sentences with *elke* non-distributively, while they may show a preference for distributive *alle*. Do these quantifiers occur in word problems? In the exploratory analysis described in the next section, we show that they occur, and that a non-adultlike interpretation is likely to lead to “wrong” answers.

3. Semantic analysis of word problems with all and every

3.1 Procedure

We surveyed word problems in second grade (‘group 4’) math materials, searching for *alle* and *elke*, to find out whether word problems exist that use these quantifiers in ways that are not yet fully acquired by 7–8 year olds. We focused on second grade, as students are 7–8 years old and are not expected to have mastered the semantic subtleties of universal quantifiers. Moreover, in second grade, students start doing multiplication and division in mathematics, for which the interpretation of collectivity/distributivity matters.

We consulted five math methods commonly used at Dutch primary schools: *Getal en Ruimte Junior*, *Alles telt Q*, *Pluspunt*, *De Wereld in Getallen* and *Rekenrijk*. We consulted all types of materials (hand book, work book, extra copies), resulting in 56 items (see the Appendix for an overview). We hand-collected all word problems containing the quantifier *alle* or *elke*. Here, we show 6 math problems that are likely to lead to the wrong answer in children:

- 3 with *elke* in which the answer key requires the quantifier to be interpreted distributively, while a cumulative or collective interpretation is possible too;
- 3 with *alle* in which the answer key requires the quantifier to be interpreted collectively, while a collective interpretation is possible too.

3.2 Word problems with *elke* ‘each’

Consider (5), for which the distributive meaning is “For each x , x is a meadow, there are 6 y ’s, y is a cow.” Given that there are 4 meadows, the target answer is 24: 4×6 cows. Following Drozd (2017: 94), the cumulative paraphrase of (5) is “There is a set of 4 meadows X and a set of 6 cows Y , and for each x in X there is a y in Y such that y stands in x and for each y in Y there is an x in X such that y stands in x .” In other words, all possible combinations of cows over meadows are allowed as long as all cows and all meadows are involved. A potential answer would be 6:

There could be 2 cows in meadow 1, 2 cows in meadow 2, 1 cow in meadow 3 and 1 cow in meadow 4, for instance.

- (5) “Ik heb 4 weilanden. In **elk** weiland staan 6 koeien. Hoeveel koeien zijn er in totaal?”
(*Alles Telt Q, Groep 4 Q-schrift*, p. 59)

“I have 4 meadows. In **each** meadow there are 6 cows. How many cows are there in total?”

Intended distributive answer: 24

Cumulative answer: 6

Problem (6) shows a similar problem as (5), but with division instead of multiplication. The intended distributive interpretation of the final sentence is “For each child *separately*, there are ... little pancakes.” On a cumulative interpretation, however, the paraphrase is “There are 6 children and ... little pancakes, and each of the children and all of the little pancakes are involved in dividing”, leading to a different answer. Note though that the verb *verdeel* ‘divide’ could aid children here in reaching a distributive interpretation.

- (6) “Reken handig. Verdeel 30 poffertjes. Verdeel over 6 kinderen. **Elk** kind krijgt ... poffertjes.”
(*Alles Telt Q, Groep 4 Blok 6*, p. 10)

“Calculate skillfully. Divide 30 little pancakes. Divide among 6 children. **Each** child gets ... little pancakes.”

Intended distributive answer: 5

Cumulative answer: 30

Finally, in (7), the accompanying illustration shows a building, multiple cameras standing at different sides of the building, and multiple pictures. The task is to enter in a table which camera angle matches with which picture.

- (7) “Zoek bij **elk** fototoestel de goede foto. Vul in.”

(*Pluspunt, Groep 4, Blok 11&12*, p. 7)

“For **each** camera, look for the correct picture. Fill in.”

Intended distributive answer: match all cameras to all pictures, one by one

Cumulative answer: match all cameras to one picture

The intended distributive interpretation of the sentence is “Find for each camera *separately* the correct picture.” The cumulative interpretation is “Find for any camera the correct picture, such that each of the cameras and the correct picture is involved”. With this interpretation, one could match all pictures to one camera – thereby arriving trivially at a collective interpretation.

In sum, there are multiple types of math problems that require *elke* be interpreted distributively to obtain the answer given in the answer key, but where a cumulative or collective reading would lead to another outcome.

3.3 Problems with *alle* ‘all’

Alle is ambiguous between a collective and a distributive interpretation. We found multiple math problems for which two interpretations are possible, even though the answer key only states one.

Problem (8) is accompanied by a picture of a book market, showing differently colored books, and a sign that says “2 euros per book”. A collective paraphrase of the final sentence is “All blue books *together* cost ... euros” while a distributive paraphrase is “All blue books *separately* cost ... euros”, leading to different answers.

- (8) “Hoeveel kosten de boeken? Vul in.

Alle blauwe boeken ... euro.” (*Pluspunt, Groep 4, Werkboek Blok 3&4*, p. 34)

“How much do the books cost? Fill in.

All blue books cost ... euros.”

Intended collective answer: 6

Distributive answer: 2

Problem (9) is accompanied by a picture with speaker A carrying a bag with 1 wood block, 1 hammer, and 1 screwdriver. With a collective interpretation (“All tools *together*”), the answer is $35+27=62$ euros. With a distributive interpretation (“All tools *separately*”), the answer is 97: 35 for the hammer, 35 for the screwdriver, and 27 for the wood.

- (9) **Spreeker A:** “Alle timmerspullen kosten 35 euro.”

Spreeker B: “En het hout kost 27 euro.”

(*Pluspunt Groep 4, Werkboek Blok 7&8*, p. 35)

Spreeker A: “All tools cost 35 euros.”

Spreeker B: “And the wood costs 27 euros.”

Intended collective interpretation: 62

Distributive interpretation: 97

Problem (10) contains a linguistic cue for collectivity, *samen* ‘together’. The problem is accompanied by pictures of the different coins. Without *samen*, the sentence is ambiguous between a collective “how much are the coins worth *together*” and a distributive interpretation “how much is each of the coins worth *separately*”, the latter leading to a list answer. Since we do not know whether 7–8 years olds reverse their preference for distributive *alle* based on the linguistic cue *samen* ‘together’, (10) as well may be problematic for school aged children.

- (10) “Op schateiland hebben ze munten van 1, 2, 4, 8, 16 en 32 florijn. Hoeveel zijn **alle** munten samen waard.” (*De Wereld in Getallen, Rekenboek 4b taken*, p. 31)

“In treasure island they have coins worth 1, 2, 4, 8, 16 and 32 florins. How much are all coins together worth?”

Intended collective answer: 63

Distributive answer: 1, 2, 4, 8, 16, 32

In sum, we found multiple instances where a collective and a distributive interpretation are available for *alle*, and either interpretation would lead to a different answer.

4. Discussion

In this explorative study, we found math problems using the quantifiers *alle* and *elke* for which an adult-like understanding is necessary to solve them in the intended way. We moreover found math problems for which multiple interpretations are available, even for adults. We have to be cautious in our conclusions, however. So far, they are solely based on a linguistic analysis of the materials. We have not been present in the classroom and do not know what is or isn't explained specifically by the teacher. Furthermore, the formatting of the problems, as well as the accompanying illustrations, may help the child arrive at the intended interpretation. Finally, sometimes linguistic cues, such as *samen* 'together', may be helpful when a particular interpretation is intended.

Our results so far give rise to two questions: First, how many potentially problematic math problems exist in math methods used at school? Are the math problems we found the exception, or the rule? Second, do children indeed struggle with the language used in these problems? We conducted pilot studies to preliminarily answer both questions. The studies suggest that (a) the problem is larger than the examples shown above, and (b) children provide alternative answers to the questions, warranting a full investigation as described in Section 5.

Concerning the number of occurrences of *alle* and *elke*, we found 86 problems with *alle* and 394 problems with *elke* on ~4400 pages of the math materials described in Section 4.1. This amounts to one word problem with one of these quantifiers on 11% of all pages, i.e., roughly, on every 10th page. While we have not yet analyzed for each of the 470 problems whether a non-adultlike interpretation results in a different answer, the total number and the proportion per page seems large enough to warrant further research.

Concerning the question whether children struggle with the problems because of the language it contains, we conducted a pilot study on the math problems with *alle*. We interviewed one 7-year-old and one 8-year-old, in which we

asked them to solve problems highly similar to the ones in Section 3. For problem (8), for instance, repeated below, both children initially gave a distributive answer: ‘each book separately costs 2 euros’. We then asked them a follow-up question, in which we hypothesized that a classmate might solve the same problem in an alternative, collective way (‘all books together cost 6 euros’); we asked them whether that classmate would also be correct in their answer. The 7-year-old answered that the classmate would be incorrect, but the 8-year-old revised his initial answer, using the plural noun form as support for the correct answer. To us, this suggests that (a) for math problems containing *alle*, children do not always give the answer provided in the answer key, and (b) children are sensitive to linguistic cues in math problems.

- (9) “Hoeveel kosten de boeken? Vul in.
Alle blauwe boeken ... euro.”
“How much do the books cost? Fill in.
All blue books cost ... euros.”

5. Research agenda

In this paper we have demonstrated a small, first step of a novel research agenda that investigates whether the language used at school matches the typical path of language development. Our analysis of word problems from Dutch math books has shown how non-adultlike interpretations of quantifiers *alle* ‘all’ and *elke* ‘each’ by children at school age may impact pupils’ understanding of the designated math operations in these problems. We contend that linguists can, and should, contribute to discussions about the richness of math language by finding out whether the language used matches the typical acquisition path.

To achieve societal impact, many further steps are necessary. We propose a mixed-methods approach in which the pilot studies discussed in Section 4 are scaled up: First, a database should be created with word problems from current math books that contain late-acquired language elements. These need to be coded on linguistically relevant properties to create a systematic overview of targeted versus non-adultlike interpretations. Second, interviews with children can help understand how they solve word problems that may be expected to cause interpretation problems. This can reveal to what extent they struggle with the language used in these problems. Teachers can furthermore be asked if they recognize the language issues. Yet another angle in the approach is to design intervention studies that can systematically investigate the effect of alternative formulations, where for instance cues such as *together* or *separately* give away the intended interpre-

tation. Finally, talking and listening to primary school teachers, math education experts, teacher trainers, publishers and curriculum developers, in the form of focus groups, can help spread awareness of the role of late acquired language in word problems to the stakeholders. The end goal is to make sure, together with experts in education, that word problems match the acquisition path.

The research agenda sketched above also offers scientific gains for the field of linguistics, in particular, detailed corpus studies of various late-acquired semantic phenomena in child-directed language and experimental studies of the same phenomena comparing children's interpretation of alternative formulations. Late language phenomena include many other quantifiers that are crucial in math language, including superlative and comparative quantifiers such as *more*, *most* and *at most* (Geurts et al. 2010). Besides quantifiers, there are late-acquired phenomena that have up to now been studied much less in acquisition: focus particles such as *even* and *only* (Heredia et al. 2022; Müller et al. 2011), modals (Cournane & Perez-Leroux 2020; van Dooren et al. 2022) and logical connectives such as disjunction and negation (Cochard et al. 2023).

Future work can investigate any of these late-acquired language phenomena in education. Here, we have focused on mathematics, but the same issues can be applied to other parts of the school curriculum (language, STEM courses). We conclude that theoretical and applied studies connecting linguistics and education in a truly interdisciplinary approach need to go hand in hand. Understanding late acquired phenomena better, and understanding why they are acquired late, we can bridge the gap between linguistic theory and educational practice.

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Appendix. Consulted math materials for group 4 (second grade)

Material	Publisher	Year
Alles telt Q – Basiswerkschrift blok 1	ThiemeMeulenhoff	2020
Alles telt Q – Basiswerkschrift blok 2	ThiemeMeulenhoff	2020
Alles telt Q – Basiswerkschrift blok 3	ThiemeMeulenhoff	2020
Alles telt Q – Basiswerkschrift blok 4	ThiemeMeulenhoff	2020
Alles telt Q – Basiswerkschrift blok 5	ThiemeMeulenhoff	2020
Alles telt Q – Basiswerkschrift blok 6	ThiemeMeulenhoff	2020
Alles telt Q – Q-schrift	ThiemeMeulenhoff	2020
Alles telt Q – Toetsschrift	ThiemeMeulenhoff	2020
De wereld in getallen 5e editie – Werkboek blok 1	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 2	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 3	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 4	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 5	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 6	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 7	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 8	Malmberg	2019
De wereld in getallen 5e editie – Werkboek blok 9	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 1	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 2	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 3	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 4	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 5	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 6	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 7	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 8	Malmberg	2019
De wereld in getallen 5e editie – Taken blok 9	Malmberg	2019
De wereld in getallen 4e editie – Bijwerkboek 4a	Malmberg	2009
De wereld in getallen 4e editie – Bijwerkboek 4b	Malmberg	2009
De wereld in getallen 4e editie – Rekenboek 4a lessen + taken	Malmberg	2009

Appendix. *(continued)*

Material	Publisher	Year
De wereld in getallen 4e editie – Rekenboek 4b lessen + taken	Malmberg	2009
Getal en Ruimte Junior – Leerwerkboek 4 blok 1	Noordhoff	2021
Getal en Ruimte Junior – Leerwerkboek 4 blok 2 en 3	Noordhoff	2021
Getal en Ruimte Junior – Leerwerkboek 4 blok 4 en 5	Noordhoff	2021
Getal en Ruimte Junior – Leerwerkboek 4 blok 6 en 7	Noordhoff	2021
Getal en Ruimte Junior – Leerwerkboek 4 blok 8 en 9	Noordhoff	2021
Getal en Ruimte Junior – Meesterwerkboek	Noordhoff	2018
Getal en Ruimte Junior – Toetsboek	Noordhoff	2017
Pluspunt – Werkboek blok 1 en 2	Malmberg	2009
Pluspunt – Werkboek blok 3 en 4	Malmberg	2009
Pluspunt – Werkboek blok 5 en 6	Malmberg	2009
Pluspunt – Werkboek blok 7 en 8	Malmberg	2009
Pluspunt – Werkboek blok 9 en 10	Malmberg	2009
Pluspunt – Werkboek blok 11 en 12	Malmberg	2009
Pluspunt – Kopieerbladen remediering	Malmberg	2009
Pluspunt – Toetsboek	Malmberg	2009
Pluspunt – Werkboek Pluspunters	Malmberg	2009
Rekenrijk – Handboek 4a	Noordhoff	2009
Rekenrijk – Handboek 4b	Noordhoff	2009
Rekenrijk – Werkboek 4a-1	Noordhoff	2009
Rekenrijk – Werkboek 4a-2	Noordhoff	2009
Rekenrijk – Werkboek 4b-1	Noordhoff	2009
Rekenrijk – Werkboek 4b-2	Noordhoff	2009
Rekenrijk – Kopieermap 1	Noordhoff	2009
Rekenrijk – Kopieermap 2	Noordhoff	2009

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